Kinematic Analysis of MTAB Robots and its integration with RoboAnalyzer Software

Ratan Sadanand O. M.
Department of Mechanical Engineering
Indian Institute of Technology Delhi
New Delhi, India
ratan.sadan@gmail.com

S. Sairaman
MTAB Engineers Pvt. Ltd
Electrical and Electronics Industrial Estate
Chennai, India
sairaman@mtabindia.com

P. H. Balaji Sah
MTAB Engineers Pvt. Ltd
Electrical and Electronics Industrial Estate
Chennai, India
balaji@mtabindia.com

G. Udhayakumar
MTAB Engineers Pvt. Ltd
Electrical and Electronics Industrial Estate
Chennai, India
udhayakumar.g@mtabindia.com

Rajeevlochana G. Chittawadigi
Department of Mechanical Engineering
Amrita School of Engineering
Bengaluru, India
rg_chittawadigi@blr.amrita.edu

Subir Kumar Saha
Department of Mechanical Engineering
Indian Institute of Technology Delhi
New Delhi, India
saha@mech.iitd.ac.in

ABSTRACT

Robotics has emerged as a research interest to find its place in various applications such as industries, automobile, space robots, health care, etc. The thrust in robotics research has resulted in increasing number of courses being introduced in the engineering curriculum. Fundamental concepts in robot mechanics are difficult to visualize using text books alone and hence, require either a physical robot or a simulation software to demonstrate the same. Effective robotics education can be achieved using serial robots and a visualization software. In this paper, affordable serial chain robots developed by MTAB are presented along with its integration with a simulation software named RoboAnalyzer. The forward and inverse kinematic analyses of the MTAB Mini and Aristo robots, and the implementation of the same in RoboAnalyzer software is also presented for comprehensive learning of the robotics topics.

Categories and Subject Descriptors
I.2.9 **[Robotics]**: Kinematics, I.3.8 **[Computer Graphics]**: Applications

General Terms
Design, Experimentation.

Keywords
Industrial robot, serial robots, DH parameters, robot kinematics, robot simulation.

1. INTRODUCTION
With the introduction of robots in diverse domains like, defense, industry, healthcare, entertainment, etc., robotics education is garnering prime focus in engineering curricula of many universities. Majority of the introductory courses on robotics focus on the mechanics and control of serial chain robots. However, this involves understanding and appreciating concepts from kinematics, matrix theory, dynamics, control theory, etc. It is often not very intuitive to learn the same using a textbook alone. An obvious way to make the teaching of robot mechanics easy and intuitive is to have a serial chain robot that can be used for explaining the geometry and architecture of the robot. By analyzing the geometry and motion of the robot, an intuitive understanding of the kinematics, dynamics and control of robots can also be acquired. An overview of the role of robotics in education is given in [1].

Industrial robots developed to perform robust tasks with higher accuracy are expensive for most of the universities to purchase and use in robotics education and research. For those universities, which have an industrial robot, it may not be possible to teach a full-fledged class with a single robot as every student may not have access to control a robot. An alternative is to have a robotics simulation software to simulate the robot motion so that many students can visualize and learn at the same time. Typically, robot manufacturers such as KUKA, FANUC, ABB, etc. have their own proprietary robot simulation software which are available for the users. These software act as an offline programming platform where a robot program is developed, simulated and tested for desired behavior or collision checks in a 3D environment and only after successful execution, robot programs are sent to robot controller to achieve the desired motion. However, these simulation software are primarily meant for use in industry and hence do not have details of Denavit-Hartenberg (DH) parameters, DH frames, forward and inverse kinematics, which are the topics of a robotics course. Moreover, they are not necessarily made available for free to the end users. Hence, there exists a need for affordable serial robots for teaching and research in robotics, and also a simulation framework for visualization of robot motion primarily meant for teaching and learning robotics concepts.

In this paper, an overview of affordable serial robots developed by MTAB [2] - namely, MTAB Mini and MTAB Aristo are presented which can be used to teach robotics and in research by universities. The integration of MTAB robots inside RoboAnalyzer [3-5], a 3D model based robot simulation software, is also presented. This paper focuses on how an affordable robot, along with a robotics learning software can be an effective teaching tool for robotics. The details and specifications of the Mini and Aristo robots are presented in Section 2, followed by their kinematic analysis in Section 3. The integration of CAD model of robots and their kinematic analysis in RoboAnalyzer is covered in Section 4 followed by the conclusions in Section 5.

2. MTAB SERIAL CHAIN ROBOTS

In this section, an overview of the MTAB robots is given. The 5-axis Mini robot and 6-axis Aristo robot have an articulated structure. To make the robot light-weight, the links of the robots are fabricated using aluminum. Photographs of Mini robot and Aristo robot are shown in Figures 1(a) and 1(b), respectively.

(a) Mini robot

(b) Aristo robot

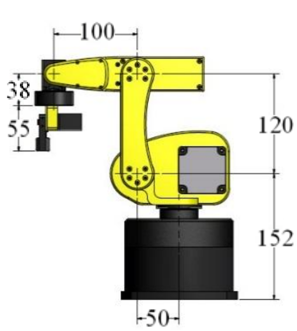
Figure 1. MTAB serial chain robots

2.1 MTAB MINI ROBOT

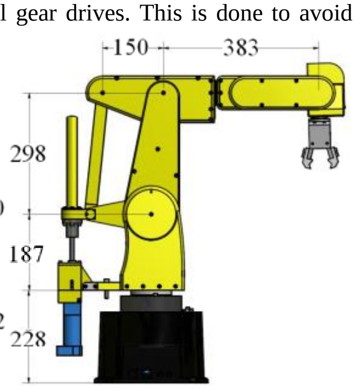
The Mini robot is a modular, table-top, five axis serial-chain robot which is developed for educational and research purposes. MTAB Mini has five revolute joints, i.e., it is a 5-axis robot having 5 degrees-of-freedom (DOF). The first three axes correspond to the position of the end-effector and the last two axes correspond to the pitch and roll motion of the wrist. The structure of Mini Robot is shown in Figure 2(a). The end-effector (EE) is a two-jaw mechanical gripper, powered by a RC servo motor. The revolute joints are driven using five hybrid stepper motors.

2.2 MTAB ARISTO ROBOT

The MTAB Aristo is a modular 6-axis manipulator designed for material and tool handling, and for specialized tasks. The reprogrammable and multifunctional Aristo is a 6 DOF wrist-partitioned robot, which can handle payloads up to 2.5 kg. The mechanical structure and the pertinent dimensions of the robot are shown in Figure 2(b). All the six joints are driven using geared DC motors. The second and third joints are actuated using a closed-loop linkage, driven using a lead screw. The linear motion of lead screw is converted into rotary motion of axes 2 and 3. This allows to impart more torque and structural rigidity at the same time. It is a cost effective solution against high precision planetary or harmonic gear or cycloidal gear drives. This is done to avoid



(a) Mini robot



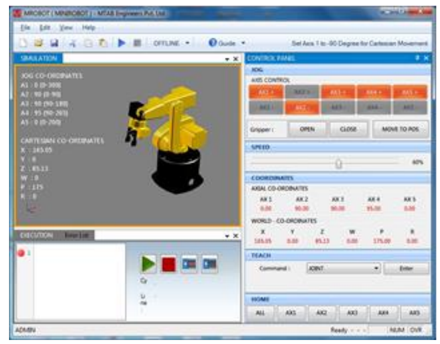
(b) Aristo robot

Figure 2. Schematic of the MTAB Robots

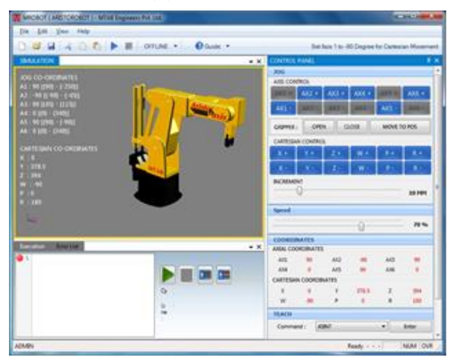
the weight of the motors, driving second and third joints, to be otherwise carried by the moving links. In fact, such an arrangement balances the weights right to the axis of joint 2.

2.3 Robot control and programming

Mini and Aristo are programmed using MROBOT software installed on a PC. The Graphical User Interface (GUI) of the MROBOT software showing the programming environment of Mini and Aristo are illustrated in Figure 3. Lead through method is used for robot programming. The MROBOT interface allows the user to teach the important points to the robot for the specified task and playback the points when the robot is used. While the Mini robot has a compact, standalone stepper motor controller, the Aristo robot is equipped with a dedicated industrial grade motion controller. The other important parameters of the robot are mentioned in Table. 1.



(a) Mini robot



(b) Aristo robot

Figure 3. GUI of MROBOT software

Table 1. Parameters of Mini and Aristo robots

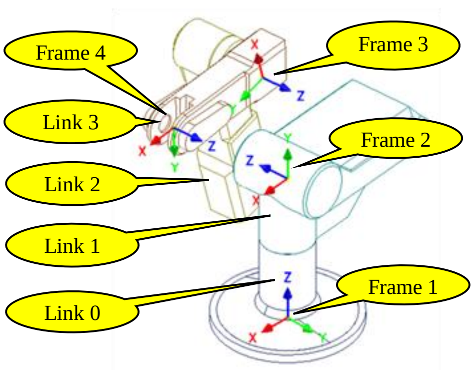
Parameters	Mini robot	Aristo robot
Payload (kg)	0.25	2.5
Robot mass (kg)	8	35
Position Feedback	Open-loop	Closed-loop
Repeatability (mm)	1	0.3
Resolution (°/mm)	1	0.1
Path type	Point-to-point	Point-to-point, linear and circular interpolation
Communication	PC USB port	Ethernet

2.4 MTAB robots in robotics education

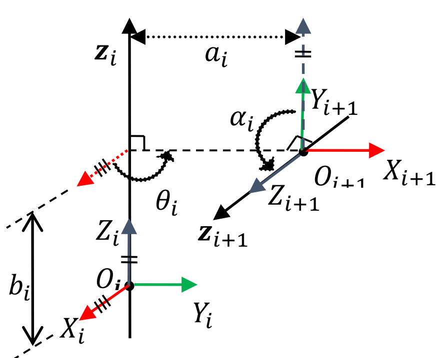
The articulated Mini and Aristo robots were developed, keeping in mind the educational applications. The affordable robots would help the students to identify the robot architecture, kinematics, programming techniques and control of an industrial serial-chain robot. The MTAB robots have been used in training for industrial operations like pick-and-place operations, parts handling, assembly operations, object manipulation, etc. Mini and Aristo robots are used in a number of universities in India and abroad for teaching robot mechanics and applications of robotics in industry.

3. KINEMATICS OF MTAB ROBOTS

The kinematic structure of a serial-chain robot is usually described using the widely accepted convention of Denavit and Hartenberg [6] parameters. A serial-chain robot, illustrated in Figure 4(a), generally consists of a number of links connected by joints of one degree-of-freedom (DOF). The joints can be either prismatic or revolute. The position and the orientation of the links can be described using the DH parameters, where each link is attached with a coordinate frame. The coordinate transformations between two successive frames can be written in terms of four DH parameters. So the complete geometry of a robot with n joints can be described using $4n$ DH parameters. The DH parameters to relate a frame F_{i+1} attached to link i with respect to frame F_i attached to link $i-1$ are defined in Table 2 and illustrated in Figure 4(b).



(a) Coordinate frames attached to links



(b) Two successive coordinate frames

Figure 4. Denavit and Hartenberg Parameters [5]

Table 2. Description of DH parameters [5]

Parameters	Description
Joint Offset (b_i)	Distance between X_i and X_{i+1} along Z_i
Joint Angle (θ_i)	Angle between X_i and X_{i+1} about Z_i
Link Length (a_i)	Distance between Z_i and Z_{i+1} along X_{i+1}
Twist Angle (α_i)	Angle between Z_i and Z_{i+1} about X_{i+1}

The relative position and orientation of a frame can be represented using a 4×4 homogenous transformation matrix (HTM), whose sub matrices represent the orientation matrix and position vector. The homogenous transformation matrix between the frames F_{i+1} and F_i, is given by [7].

$$\mathbf{T}_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & b_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

The position and orientation of the n^{th} joint with respect to the first frame (which is usually fixed) is then given by successive multiplications of the HTMs as

$$\mathbf{T} = \mathbf{T}_1\mathbf{T}_2 \ldots \mathbf{T}_i \ldots \mathbf{T}_n \tag{2}$$

Note that robot kinematics consists of two major analyses – forward kinematics and inverse kinematics. Forward kinematics involves the determination of the position and orientation of the EE when the values of the joint variables are known. It involves the determination of the HTM, **T** of the EE, given by Equation 2. This is straightforward yields only one solution. Inverse kinematics, on the other hand, deals with the determination of joint variables for a given set of EE position and orientation. Unlike forward kinematics, inverse kinematics problem can lead to multiple solutions, or no solutions at all.

To perform forward and inverse kinematic analyses of MTAB robots, the DH parameters of the robots should be known. Though some of these parameters can be retrieved from drawing/specs of the robots, to get all the parameters, an analytical method proposed in [8, 9] was used. Details of the DH parameters extraction are given in Section 4.2.

3.1 Forward kinematics

Once the DH parameters are known, their values can be used in Equation 1 to find the HTM of the coordinate frame, i.e., the DH frame, on the first link with respect to the DH frame on the base link. Similarly, HTM between the DH frames on consecutive links of the robot can be determined. Equation 2 can then be used to find the HTM of the end-effector. Note that since all the joints are revolute in Mini and Aristo robots, the expression derived from Equation 2 will require joint angles as variables. For any given set of joint angle values (θ_i's), the end-effector position and orientation can be determined, thus solving the problem of forward kinematics.

It is pointed out here that the MROBOT software can take the joint angles of Mini and Aristo as input and accordingly, the end-effector position and orientation, as well as the Yaw, Pitch, and Roll (YPR) are displayed and the 3D model gets updated. However, the details of the individual HTMs and the DH frames are not available. These will be achieved using the RoboAnalyzer software, as explained in Section 4.

3.2 Inverse kinematics

The inverse kinematics analysis is more involved as it requires the solution of multiple equations with non-linear and transcendental terms. The existence of multiple solutions makes the inverse kinematic analysis more challenging when compared to the forward kinematics analysis. There are different ways to solve inverse kinematics for different robot architectures. Most of the industrial robots are 6 DOF wrist-partitioned robots which have analytical methods to determine eight possible sets of solutions. However, for a different DOF or robot having different architecture, inverse kinematics solutions have to be either determined using geometric methods or using numerical or iterative process. The inverse kinematics for Mini and Aristo robots are presented next.

3.2.1 Inverse kinematics of MTAB Mini

As per existing literature, the end-effector transformation matrix of the Mini robot can be symbolically represented as follows:

$$\mathbf{T} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Here, (p_x, p_y, p_z) represent the position vector and (n_x, n_y, n_z), (o_x, o_y, o_z), and (a_x, a_y, a_z) are the normal, orientation and approach vectors that denote the orientation of the end-effector [8]. Then the inverse kinematics solution can be written in terms of the above variables and solved using an analytical approach. The values for the terms of the HTM of EE as given in Equation 3 were calculated symbolically in terms of the joint variables and other DH parameters. Using the values of the HTM given in Equation 3, the value of $\theta_1$is calculated first. We define the quantities below, which are needed to find θ_1:

$$A = p_x - a_5 n_x \quad (4)$$

$$B = p_y - a_5 n_y \quad (5)$$

$$C = \frac{b_4}{\sqrt{A^2 + B^2}} \quad (6)$$

Then θ_1 is given by:

$$\theta_1 = Atan2\left(C, \sqrt{1 - C^2} \right) + Atan2(B, A) \quad (7)$$

Using the above value of θ_1, the value of θ_5 is then found as:

$$D = n_x \sin\theta_1 - n_y \cos\theta_1 \quad (8)$$

$$\theta_5 = Atan2\left(D, -\sqrt{1 - D^2} \right) \quad (9)$$

Using the value of joint angle θ_1, the value of θ_{234}, where $\theta_{234} = \theta_2 + \theta_3 + \theta_4$, is evaluated as:

$$\theta_{234} = Atan2\left(a_x \cos\theta_1 + a_y \sin\theta_1, a_z \right) \quad (10)$$

Next, to solve for the angle θ_3, the following quantities are defined:

$$E = p_x \cos\theta_1 + p_y \sin\theta_1 - a_1 - b_5 \sin\theta_{234} - a_5 \cos\theta_5 \cos\theta_{234} \quad (11)$$

$$F = p_z - b_1 + b_5 \cos\theta_{234} - a_5 \cos\theta_5 \sin\theta_{234} \quad (12)$$

$$G = \frac{E^2 + F^2 - {a_2}^2 - {a_3}^2}{2a_2 a_3} \quad (13)$$

$$\theta_3 = Atan2\left(\mp\sqrt{1 - G^2}, G \right) \quad (14)$$

From Equation 14, it is clear that θ_3 has two possible solutions. Having found θ_3, the following equations are used to calculate θ_2:

$$J = (a_2 + a_3 \cos\theta_3)F - (a_3 \sin\theta_3)E \quad (15)$$

$$K = (a_2 + a_3 \cos\theta_3)E - (a_3 \sin\theta_3)F \quad (16)$$

Based on the two values of θ_3, two values for θ_2 can be determined as:

$$\theta_2 = Atan2(J, K) \quad (17)$$

Finally, θ_4 is found as:

$$\theta_4 = \theta_{234} - \theta_2 - \theta_3 \quad (18)$$

Thus MTAB Mini robot has two possible inverse kinematics solutions. This means that the two sets of the five joint angles can be determined that correspond to the same position and orientation.

3.2.2 Inverse kinematics of MTAB Aristo

MTAB Aristo being a 6 DOF wrist-partitioned robot, the position and orientation problems in its inverse kinematics can be decoupled and solved. The solution proposed in [4, 7, 11] was used to obtain up to 8 possible solutions for a given position and orientation. Note that for some input value of the HTM, lesser than 8 or no solutions can also exist.

4. MTAB ROBOTS IN ROBOANALYZER

In this section, the methodology used to integrate MTAB robots in RoboAnalyzer software has been explained. The objective behind integrating the robot models in RoboAnalyzer software was twofold.

1. Able to visualize the robot's motion in the absence of the physical robot. This is a very useful feature because the students can use it outside the laboratory, where they do not have access to the real robot even if the institute has one.
2. Visualize the DH parameters, frames, etc. which cannot be placed on the real robot. It helps in understanding the concept of DH parameters and the related transformations which are not very intuitive.

4.1 RoboAnalyzer: An overview

RoboAnalyzer allows the users to perform kinematic and dynamic analyses of serial-chain robots, using skeleton model of a robot for a given set of DH parameters or by using CAD models of some standard robots. It has separate modules for DH parameter visualization, forward and inverse kinematics, forward and inverse dynamics, graph plotting, etc. There is also a Virtual Robot Module (VRM) in RoboAnalyzer that allows the joint-level and Cartesian-level motion of the robots models. More details on the implementation of RoboAnalyzer can be found in [3-5].

4.2 Import of CAD models of robots

To integrate MTAB robots, or any serial-chain robots for that matter, in RoboAnalyzer, the CAD model of each of the links of the robot should be modified and stored in a particular way, as outlined in [10]. In brief, the DH parameters were extracted from the CAD assembly model of the robot using an analytical methodology proposed in [9]. This is illustrated in Figure 5.

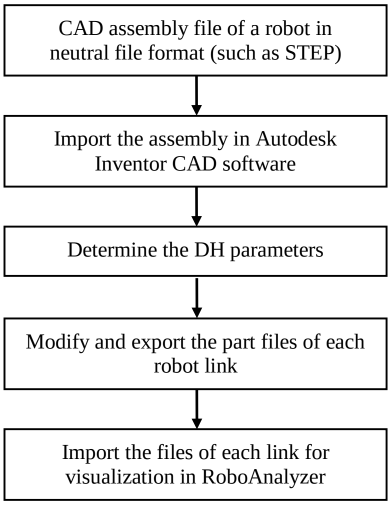


Figure 5. Integration of robot CAD model in RoboAnalyzer [8]

Accordingly, the CAD assembly of MTAB robots were imported in Autodesk Inventor and the DH parameters were extracted. As an illustration, the CAD assembly model of the MTAB Aristo robot inside Autodesk Inventor along with the DH parameters extraction add-in are shown in Figure 6. Note here that the screws of joints 2 and 3 are not shown in Figure 6 as they were not imported due to formation of closed-chain in the robot which cannot be handled by RoboAnalyzer. However, its motions were directly accounted for in the joints 2 and 3 of RoboAnalyzer model of Aristo. Hence, the kinematic analyses in RoboAnalyzer are true representations of the real motion by the actual robots.

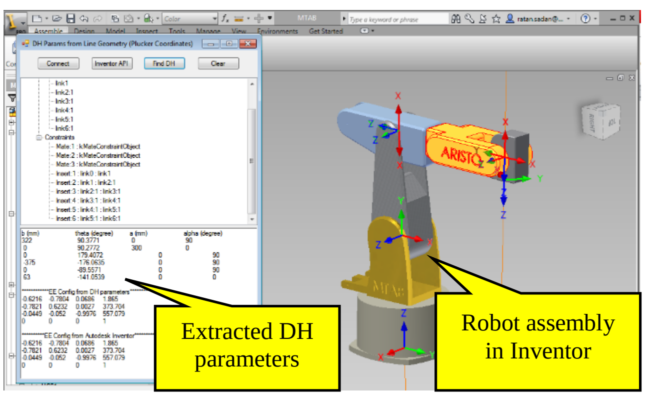


Figure 6. DH parameter extraction add-in for Autodesk Inventor

Once the DH parameters were extracted, the CAD files of the links were saved in STL format. By using an XML file in which the robot geometry and mass-inertia properties can be specified, the robot model was imported in RoboAnalyzer to leverage different features of RoboAnalyzer software.

4.3 Forward kinematics in RoboAnalyzer

The forward kinematics analysis of Mini and Aristo robots was implemented calculating the HTMs for all the sets of joint angles between the initial and final joint configurations that are specified by a user. The joint trajectories for the forward kinematic motion can also be specified by the user in RoboAnalyzer. Commonly used joint trajectories like cubic, cosine, quintic, etc. are available in RoboAnalyzer for quick analysis, which can be selected by the user. The forward kinematics motion of the Aristo robot, along with the trace of the EE is shown in Figure 7.

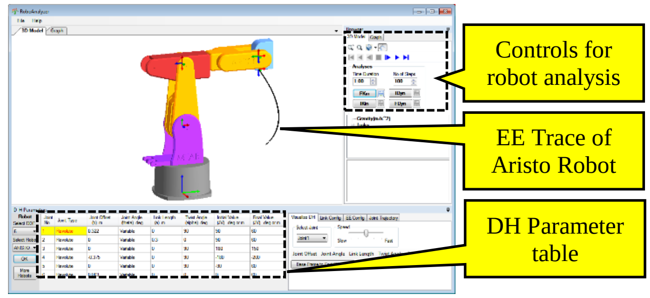


Figure 7. Forward kinematic motion of Aristo robot in RoboAnalyzer

One advantage of the visualization environment of RoboAnalyzer is that it displays the DH frames attached to each of the links and displays the frame transformations associated with each of the DH parameters. This is a unique feature that is not usually found in the typically available robot control software, commercial or otherwise. The HTMs for all the DH frames as well as the EE can also be determined using the RoboAnalyzer.

RoboAnalyzer also has provision for plotting the results of the kinematic analysis, using its graph plotting module. The values of end-effector coordinates *X, Y,* and *Z* of the MTAB Aristo robot for the motion shown in Figure 7 is plotted in RoboAnalyzer and shown in Figure 8.

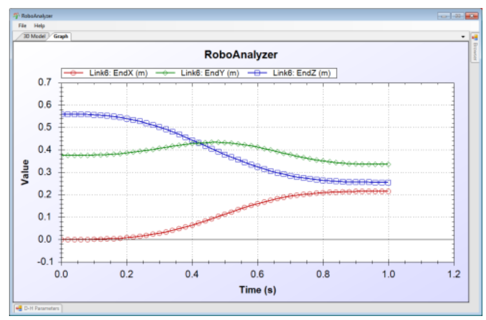


Figure 8. Plots of EE coordinates of MTAB Aristo

4.4 Inverse kinematics in RoboAnalyzer

The inverse kinematics analysis for the Mini robot presented in Section 3.2.1 was implemented in the inverse kinematics module of RoboAnalyzer. The users can calculate the two inverse kinematics solutions for a valid position and orientation of the end-effector of MTAB Mini and visualize them in RoboAnalyzer. Figure 9 shows the two possible solutions of the MTAB Mini robot for a given pose of EE.

Once the values of the joint angles have been determined, the solutions can also be visualized by performing the forward kinematics with the values of the joint angles at hand. Using the inverse kinematics module, the existence of multiple solutions for the inverse kinematics of Mini robot can be demonstrated easily using RoboAnalyzer. The same can be done using the actual robot, if available. Similarly, the inverse kinematics of MTAB Aristo robot can be solved and visualized in RoboAnalyzer, where up to 8 possible solutions can be obtained for the robot.

Often, it is not straightforward to understand and visualize multiple solutions of inverse kinematics analysis using a robot. A robot will select only one single solution from the multiple solutions while performing the task. But for robotics education, it

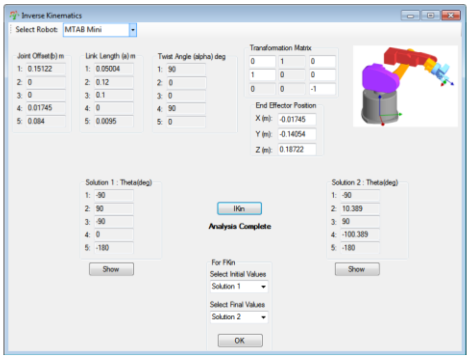


Figure 9. Inverse kinematics of MTAB Mini in RoboAnalyzer

is necessary to visualize the multiple inverse kinematics solution, for which a robotics teaching software will be convenient. RoboAnalyzer thus fulfils this aspect by allowing visualization and inspection of all possible solutions for the inverse kinematics problem.

4.5 MTAB robots in Virtual Robots Module

The Virtual Robots Module (VRM) of RoboAnalyzer allows joint-level motion of serial-chain robots, and Cartesian-level motion for 6-axis wrist-partitioned robots. The teach pendant like interface allows a user to control the motion of the robot, similar to the way in which an actual robot is controlled. The user-interface also features an option to view the end-effector configuration in terms of the HTM, as well as the position coordinates and the Euler angles, i.e., yaw, pitch and roll angles. The CAD models of Mini and Aristo robots were also integrated in the VRM. Aristo robot in VRM is shown in Figure 10.

By having the robot model in a virtual environment, the motion of the robot can be simulated and studied, before the programming is done in the actual robot. This would help the students to avoid mistakes while handling the actual robot. Even in the absence of the robot, the students can simulate the motion of the robot, thus making it a good tool for education.

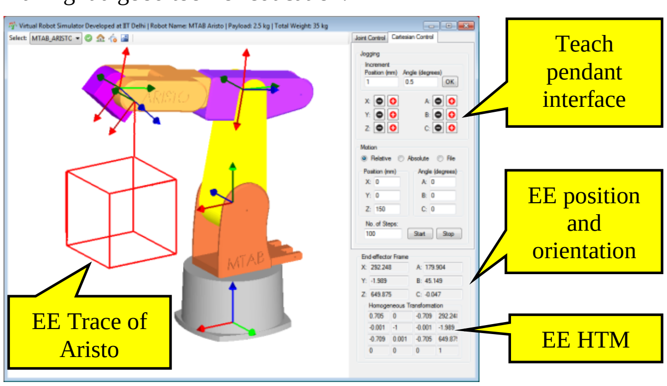


Figure 10. Cartesian motion of MTAB Aristo in Virtual Robots Module of RoboAnalyzer

5. CONCLUSIONS

The affordable serial-chain robots – Mini and Aristo developed by MTAB can be used for teaching robotics and in robotics research. They aid in the learning of robot mechanics, control, and programming of serial-chain robots for common applications like material handling, assembly, etc. Although having an actual robot can help in learning the applications, the concepts of robot mechanics must be learnt first, which may not be intuitive. Along with the robot control software, the inclusion of the robot models in RoboAnalyzer software will help in gaining an intuitive understanding of robot kinematics. The Virtual Robots Module of the RoboAnalyzer also aid in simulating the robots in a virtual environment, even in the absence of the actual robot. This would aid towards the effective use of MTAB robots in education, where the concepts of kinematics can be learnt using the RoboAnalyzer software and then later implemented in practice using the MTAB robots. RoboAnalyzer is freely available for academic use from http://www.roboanalyzer.com .

6. REFERENCES

[1] Miller, D., Nourbaksh, I., R., and Siegwart, R., 2008. Robots for education. *Springer Handbook of Robotics*, Springer Berlin-Heidelberg, 1288-1301.

[2] MTAB Robotics, 2015. http://www.mtabindia.com/robotics/index.htm as accessed on 31st January 2015.

[3] Rajeevlochana, C. G., and Saha, S. K., 2011. RoboAnalyzer: 3D Model Based Robotic Learning Software. *International Conference on Multi Body Dynamics*, 3-13.

[4] Bahuguna, J., Chittawadigi, R. G., and Saha, S. K., 2013. Teaching and Learning of Robot Kinematics Using RoboAnalyzer Software. *International Conference on Advances in Robotics*.

[5] Sadanand, R., Chittawadigi, R. G., and Saha, S. K., 2013. Virtual Robot Simulation in RoboAnalyzer. *1st International and 16th National Conference on Machines and Mechanisms*.

[6] Denavit, J., and Hartenberg, R. S., 1955. A Kinematic Notation for Lower pair mechanisms based on Matrices. *ASME Journal of Applied Mechanisms*, 22(2), 215 – 221.

[7] Saha, S. K., 2014. *Introduction to Robotics*, 2nd ed., McGraw-Hill Higher Education, New Delhi.

[8] Xu, D., Acosta, C. A., Gan, J. Q., and Hu, H., 2005. An Analysis of the Inverse Kinematics for a 5-DOF Manipulator, *International Journal of Automation and Computing*, 114-124.

[9] Chittawaidi, R.G., 2013. Unified Framework for Geometric Modeling, Animation and Collision Detection of Serial Robots. *M.S.(Research) Thesis, Department of Mechanical Engineering, Indian Institute of Technology Delhi*, New Delhi, India.

[10] Rajeevlochana, C.G., Saha, S.K., and Kumar, S., 2012. Automatic Extraction of DH Parameters of Serial Manipulators using Line Geometry, *The 2nd Joint International Conference on Multibody System Dynamics (IMSD)*.

[11] Angeles, J., 2002. *Fundamentals of Robotics Mechanical Systems*, vol. 2. Springer , New York.