Kinematic Analysis of Mechanisms using Velocity and Acceleration Diagrams (VAD) Module in MechAnalyzer Software

Sachin Kumar Verma[*], Rohit Kumar[ˀ], Rajeevlochana G. Chittawadigi[#], Subir Kumar Saha[†]

[*,ˀ] Department of Mechanical Engineering
National Institute of Technology Karnataka
Surathkal, Mangalore - 575 025. India
[*]sachinv404@gmail.com, [ˀ]rohit.kumar.in@ieee.org

[#] Department of Mechanical Engineering
Amrita Vishwa Vidyapeetham, Amrita University
Kasavanahalli, Bengaluru - 560 035. India
rg_chittawadigi@blr.amrita.edu

[†] Department of Mechanical Engineering
Indian Institute of Technology Delhi
Hauz Khas, New Delhi - 110 016. India
saha@mech.iitd.ac.in

ABSTRACT

Kinematic analysis of mechanisms is a precursor to their dynamic analysis and hence is a very important component in the courses related to Machine Design and Theory of Machines. There are various methods like analytical, numerical, and graphical for kinematic analysis of mechanisms. While teaching to undergraduate students graphical methods are emphasized in the curriculum as the students find them relatively easy to understand. Graphical method includes drawing of position of links followed by the velocity and acceleration diagrams in vector-loop form yielding velocity and acceleration polygons. Drawing these polygons is time consuming and becomes tedious for different orientations of links. A computer based approach can certainly be a useful tool in this matter. There are various software packages available which seem to be helpful in forward kinematic analysis. However, in the best knowledge of the authors, none of them draw velocity and acceleration polygons. In this paper, a module to draw Velocity and Acceleration Diagrams or VAD module is presented. It was developed as a part of MechAnalyzer software, a 3D model based mechanism learning software. The VAD module is helpful in forward kinematic analysis of various planar mechanisms preloaded in it, and draws position, velocity, and acceleration polygons. It has an interesting feature of animating the drawing of polygons in the way they are drawn by a teacher on the board of a classroom.

Keywords: Simulation, Forward kinematic analysis, Vector diagrams, Planar mechanisms.

1. INTRODUCTION

Multibody dynamics is a vast area and starts with the kinematic analysis of basic planar mechanisms. Understanding of basic concepts related to kinematic analysis is of prime concern. There are several methods available for kinematic analysis of mechanisms such as analytical, numerical iterative, and graphical methods. Analytical methods involve formulation of equations of motion in terms of unknown parameters obtained through geometric relationships between the links and joints of a mechanism. Numerical iterative methods are typically used for systems which do not have a closed form solution. Graphical methods use geometric methods to find the relationships between different input and output parameters. The graphical methods which are widely used require the drawing of the position of links, followed by the velocity and acceleration diagrams in the closed-loop form yielding velocity and acceleration polygons. In teaching the kinematic analysis of mechanisms, these methods are more intuitive over the others as students find it relatively easy to understand because of the involvement of geometry. Apart from this, it also has visual essence which helps students to correlate position, velocity, and acceleration of links. Once students are familiar with the graphical method, they can attempt analytical methods with better understanding and confidence.

The major drawback in this method is, however, the time consumed to draw the position, velocity, and acceleration polygons. Moreover, drawing them for different link orientations is very tiresome. Drawing velocity and acceleration polygons during teaching consumes a lot of time which necessitates the use of simulation software. There are various software developed by various organizations which can be used to teach mechanisms. A few of these are MD Adams [5], SAM [10], GIM [7], Universal Mechanisms [13], MMtool [11] and Linkage [3], of which some are available for free while others are paid. A free software was developed by the authors in the name of MechAnalyzer [2, 4]. An overview of the available software for mechanisms and how MechAnalyzer is better at some of the aspects related to teaching are given in [2, 4].

After doing a comparative study of the available software, it has been found, to the best of the knowledge of the authors, that none of them have a provision to draw position, velocity, and acceleration diagrams. The authors have developed a new module named Velocity and Acceleration Diagrams (VAD) module, as a part of continued development of MechAnalyzer. It serves the purpose of forward kinematic analysis of various planar mechanisms like four-bar, slider-crank, Whitworth quick return, crank and slotted lever, elliptical trammel, and scotch-yoke mechanism, which are available as default mechanisms. Position, velocity, and acceleration vector polygons are shown to a user for default values of link length and orientations. These polygons are redrawn spontaneously for any change in the input parameters. Results obtained after analysis of mechanism are shown to the user. Another interesting feature of the VAD module is that it animates the vector polygons in the way they are drawn on a black board, and hence helps teachers for effective teaching and improves students' ability to learn.

This paper is organized in the following manner: In Section 2, a brief introduction of different versions of MechAnalyzer software is presented, and Section 3 is dedicated to the description of VAD module and its specifications. Methodology for forward kinematic analysis of slider-crank mechanism is presented in Section 4. The validation of the results with a commercial modelling software, namely, Autodesk Inventor Professional (student version), is covered in Section 5, followed by the future scope and conclusions in Section 6 and 7, respectively.

2. AN OVERVIEW OF MECHANALYZER

MechAnalyzer is a 3D model based mechanism learning software. It was developed with the purpose of teaching and learning concepts related to mechanisms and multibody dynamics more effectively. Versions 1 and 2 of MechAnalyzer [2] perform forward kinematic analysis of planar mechanisms which show animation of the motion, and plot graphs of the results. It has preloaded planar mechanisms like four-bar, slider-crank and five-bar mechanisms so that a student could select anyone of them and without much hassle, get started to analyze a mechanism. It was developed in Microsoft Visual Studio using Visual C# programming language. The graphics were implemented using OpenTK (a wrapper for OpenGL) and plots were drawn using ZedGraph library. A screenshot of the versions 1 and 2 of MechAnalyzer is shown in Figure 1.

To have similarity with another software called RoboAnalyzer [8] developed by last two authors to teach robotics, MechAnalyzer Version 3 [4] was developed with a new interface and with various improved features. More mechanisms were loaded in it, e.g., Whitworth quick return, double slider, four-bar quick return, cam, gear, steering, pantograph, and wiper mechanism. Different variants of a mechanism were also loaded if they existed. This version lacked the graph plots though which was taken up in version 4. The user interface of MechAnalyzer Version 3 is shown in Figure 2.

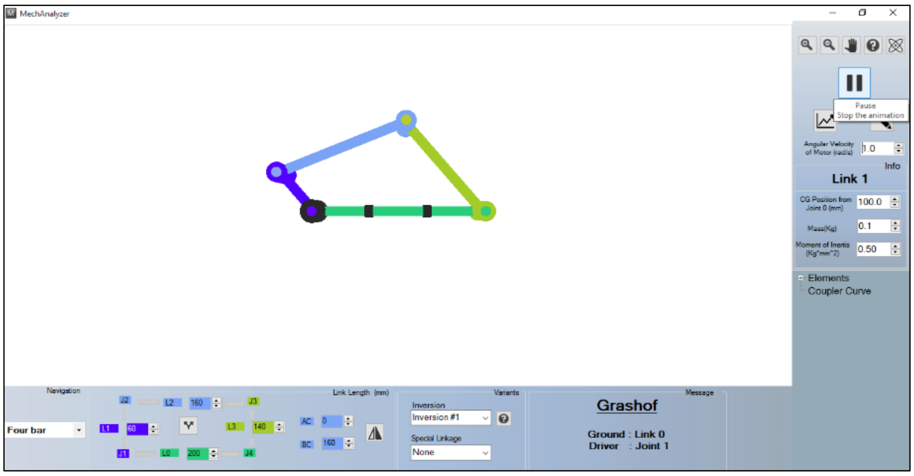


Figure 1: User Interface of MechAnalyzer Versions 1 and 2

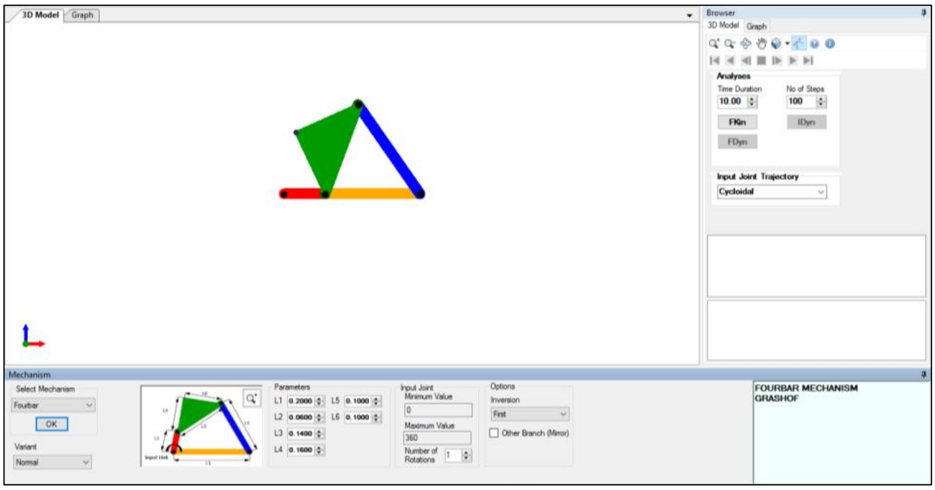


Figure 2: User Interface of MechAnalyzer Version 3

In Version 4, VAD module was also added along with the plots for the kinematic analysis of the preloaded mechanisms present in Version 3. In this paper, the implementation details of VAD module alone are discussed and not the plots available in Version 4.

3. OVERVIEW OF THE VAD MODULE

The VAD module has been developed as a part of MechAnalyzer software. It pops up as a separate window when the button provided on the user interface is clicked. It has been developed using programming language Visual C# in Microsoft Visual Studio as a Windows Form Application. For drawing vector diagrams, Zedgraph library was used, which is an open source library for drawing 2D plots in C#. Primary objectives of the VAD module that were considered during its development are:

- It should be able to perform the forward kinematic analysis of common planar mechanisms encountered in the course of Theory of Machines.
- It should aid in teaching the graphical method of analyzing planar mechanisms.
- It should be able to draw position, velocity and acceleration diagrams in order to avoid manual drawing on the black board while teaching.
- It should animate drawing of vector diagrams to elucidate the procedure of how they are drawn to make learning of mechanism effective for the students.

The VAD module performs forward kinematic analysis of four-bar, slider-crank, Whitworth quick return, crank and slotted lever, elliptical trammel and scotch-yoke mechanism, all of which have one degree-of-freedom (DOF). It takes the input in the form of link lengths, input crank angle, and crank's angular velocity. Results obtained after analysis of a mechanism are shown in the form of all unknown parameters such as angle of each link (with respect to horizontal axis), angular velocity, and angular acceleration of each link (except crank). Vector diagrams for position, velocity and acceleration are shown to the user which are involved in the analysis. There is an interesting feature provided in the VAD module which animates vector diagrams to mimic the drawing by a teacher. Vector diagrams are shown for default values and gets changed when input parameters are changed. Graphical user interface of the VAD module is shown in Figure 3. The user interface window is partitioned into three major parts, which are vector diagrams panel, control panel, and results and message box panel.

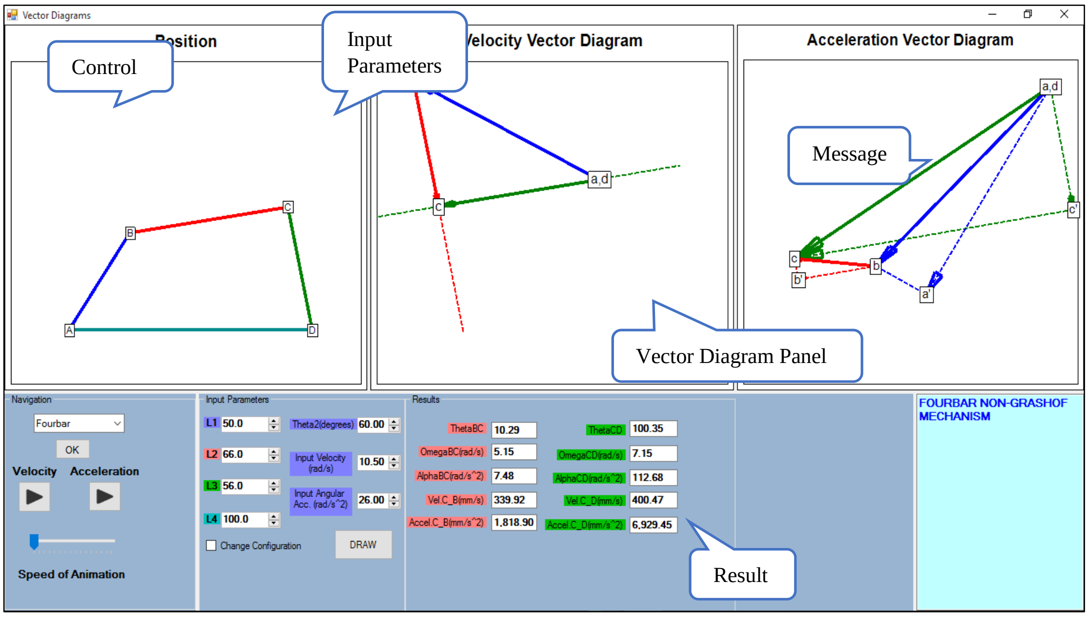


Figure 3: User Interface of the VAD Module

The vector diagram panel consists of three boxes which are used to draw position, velocity, and acceleration vector diagrams and also their animation in the way they are drawn. Each box is created using a ZedGraph entity. Control panel is further subdivided into two subparts, namely, a navigation box and input parameters box. Navigation box consists of control to select desired mechanism from dropdown menu and buttons to play or pause the animation. There is a track bar provided for varying the speed of animation. The input parameters box is used to provide parameters for a mechanism. Draw button is provided to redraw the vector diagrams for new parameters. Different orientations (if available) can also be drawn for a mechanism. Results panel is provided to show analysis results and, a message box to display the name of the mechanism which is currently been analyzed and any error (if occurred). A uniform color has been followed throughout for drawing of vector diagrams to maintain consistency, for example, a blue colored link in position diagram has the same color for its velocity and acceleration vectors.

4. METHODOLOGY OF ANALYSES

Graphical and analytical methods were used for the formulation of forward kinematics. After getting the link lengths, crank angle, and angular velocity, the angles of other joints were calculated by solving algebraic equations which were obtained through trigonometric relations. These were mainly derived using the formulation in [6]. However any standard text books like [1], [9], and [12] can also be used. Detailed explanation is divided in two subsections on position analysis, and velocity and acceleration analyses. Here, the analysis of a slider-crank mechanism is explained in detail, as it has both revolute as well as prismatic joints.

4.1. POSITION ANALYSIS OF A SLIDER-CRANK MECHANISM

Position analysis of a slider-crank mechanism is shown here using analytical method. The algebraic equations obtained through trigonometric relationship were solved for unknown joint angles or values. These results allow one to find the position and the orientation of each link. The nomenclature used to define the links and angles is shown in Figure 4.

Consider the mechanism to have links named Link 1, Link 2 and Link 3, and having link position vectors $\mathbf{r_1}$, $\mathbf{r_2}$, and $\mathbf{r_3}$ having magnitudes r_1, r_2, and r_3 respectively. Slider is named as Link 4 which is connected to the ground link, i.e. Link 1, through prismatic joint and is free to undergo reciprocating motion.

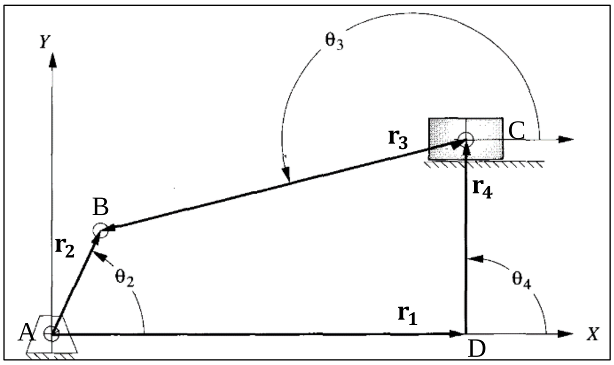


Figure 4: Nomenclature of links and angles for slider-crank mechanism

Slider Link 4 has an offset from the ground Link 1 whose vector is $\mathbf{r_4}$ with magnitude r_4. Let the input crank angle be θ_2 and the rest of the angles are shown in the figure. The vector loop equation is given by

$$\boldsymbol{r_2} + \boldsymbol{r_3} = \boldsymbol{r_1} + \boldsymbol{r_4} \tag{1}$$

Equation (1) can be arranged in the components form as

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 = r_1 \tag{2}$$
$$r_2 \sin\theta_2 + r_3 \sin\theta_3 = r_4 \tag{3}$$

From Equation (3), the orientation of Link 3 can be obtained as

$$\theta_3 = \sin^{-1}\frac{r_4 - r_2 \sin\theta_2}{r_3} \tag{4}$$

The Slider (Link 4) position can be found using Equation (2) as

$$r_3 = \frac{r_1 - r_2 \cos\theta_2}{\cos\theta_3} \tag{5}$$

After calculating θ_3 and r_3, an equivalent position diagram can be drawn which is shown in Figure 5.

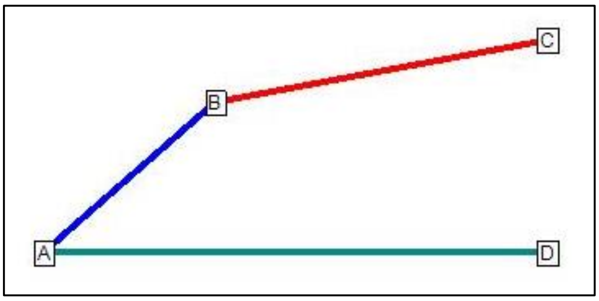


Figure 5: Vector diagram for the position of slider-crank mechanism

4.2. VELOCITY ANALYSIS OF THE SLIDER-CRANK MECHANISM

In the graphical method, the orientation of each link obtained from the position analysis is used for the velocity analysis. If a link has a rotary motion with respect to any other link, the direction of motion of the link and hence its velocity vector will be perpendicular to the link. On the other hand, if the motion is translatory, the direction of its velocity vector will be along the direction of the motion. In the case of the slider-crank mechanism at hand, a vector for Link 2 (crank) is drawn on a suitable scale for which magnitude of velocity and direction are known from the input parameters. The direction of the velocity vector of Link 3 i.e., connecting rod, is perpendicular to it. The direction of velocity vector of Link 4 (slider) is along its translatory motion. The intersection point can be determined for these two directions and hence a vector polygon is obtained. Sides of the polygon represent velocity vectors which correspond to the direction of the velocity, and by mapping its magnitude to the original scale, the linear velocity of the links are calculated. More information on the graphical methods can be found in [6], [1], [9], and [12].

For the implementation of MechAnalyzer, the magnitude of the linear velocity at the endpoint of each link was determined to get the velocity polygon. Product of the magnitude of angular velocity and the length of the link gives the linear velocity of the end point of each link. Referring to Figures 4 and 5, let the angular velocity of Link 2 be ω_2 and its length be r_2 ($\equiv|\overrightarrow{\mathbf{AB}}|$). The linear velocity of point B with respect to point A will be:

$$v_2 = \omega_2 \, r_2 \tag{6}$$

First draw velocity vector $\overrightarrow{\mathbf{ab}}$ for Link 2 (Figure 6) normal to AB (Figure 5) with magnitude as v_2. Now, draw the velocity vector $\overrightarrow{\mathbf{bc}}$ and $\overrightarrow{\mathbf{dc}}$ normal to $\overrightarrow{\mathbf{BC}}$ and $\overrightarrow{\mathbf{DC}}$**,** respectively, in order to obtain the velocity polygon. Note that point D is drawn always below point C. Magnitudes of $\overrightarrow{\mathbf{bc}}$ and $\overrightarrow{\mathbf{dc}}$ are equal to the linear velocity of Link 3 and Link 4, respectively. Velocity vector diagram obtained in this way for slider crank mechanism is shown in Figure 6. Let the angular velocity for Link 3 be ω_3 with its length as r_3 ($\equiv|\overrightarrow{\mathbf{BC}}|$). The angular velocity is then given by

$$\omega_3 = \frac{|\overrightarrow{\boldsymbol{bc}}|}{r_3} \tag{7}$$

Linear velocity of Link 4 (slider) can be obtained as

$$v_4 = |\overrightarrow{\boldsymbol{dc}}| \tag{8}$$

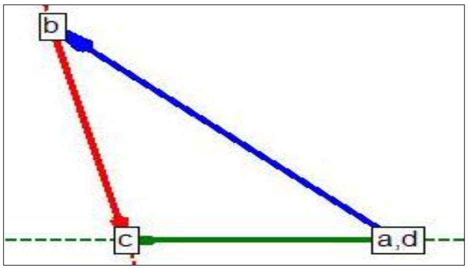


Figure 6: Velocity vector diagram for slider crank mechanism

4.3. ACCELERATION ANALYSIS OF THE SLIDER-CRANK MECHANISM

In case of the acceleration analysis, there are two components of the linear acceleration of each link. They are radial and tangential. Let a_2^r and a_2^t be radial and tangential components of acceleration respectively for Link 2. Angular acceleration α_2 of Link 2 will be taken as the input from user. Its radial and tangential components are determined using

$$a_2^r = \omega_2^2 r_2 \tag{9}$$
$$a_2^t = \alpha_2 r_2 \tag{10}$$

So resultant acceleration of Link 2 can be obtained as

$$a_2 = \sqrt{({a_2^r}^2 + {a_2^t}^2)} \tag{11}$$

Link 4 (slider) has only linear velocity so it will have only tangential acceleration component. The radial acceleration component for Link 3 will be:

$$a_3^r = \omega_3^2 r_3 \tag{12}$$

The implementation of acceleration vector diagram is shown in Figure 7. The vectors $\overrightarrow{\mathbf{aa'}}$ and $\overrightarrow{\mathbf{ab'}}$ are drawn along and normal to $\overrightarrow{\mathbf{AB}}$ with magnitude a_2^r and a_2^t, respectively. Similarly vector $\overrightarrow{\mathbf{bb'}}$ is drawn along $\overrightarrow{\mathbf{BC}}$ with magnitude a_3^r. Vector $\overrightarrow{\mathbf{b'c}}$ and $\overrightarrow{\mathbf{cd}}$ are drawn normal to $\overrightarrow{\mathbf{BC}}$ and along $\overrightarrow{\mathbf{DC}}$, respectively, to get a polygon. Acceleration vector diagram obtained in this way is shown in Figure 7.

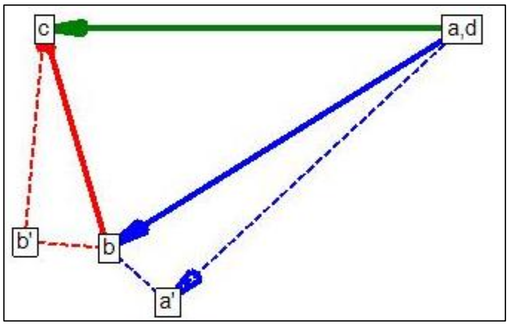


Figure 7: Acceleration vector diagram for slider-crank mechanism

The linear acceleration of slider (Link 4) is obtained using

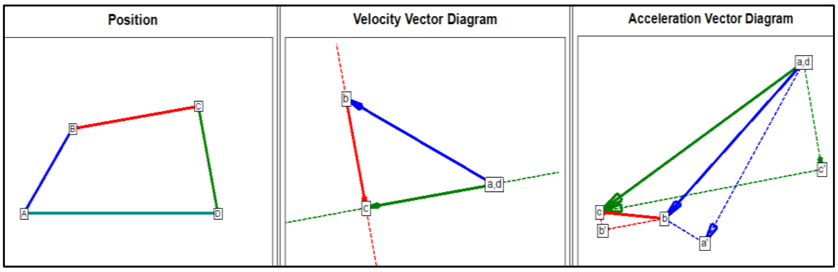
$$a_4 = |\overrightarrow{\mathbf{dc}}| \tag{13}$$

The linear acceleration of endpoint of Link 3 can be obtained as

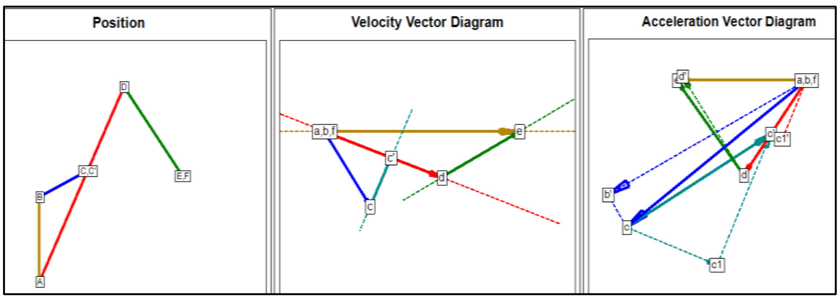
$$a_3 = |\overrightarrow{\mathbf{bc}}| \tag{14}$$

4.4. PLANAR MECHANISMS

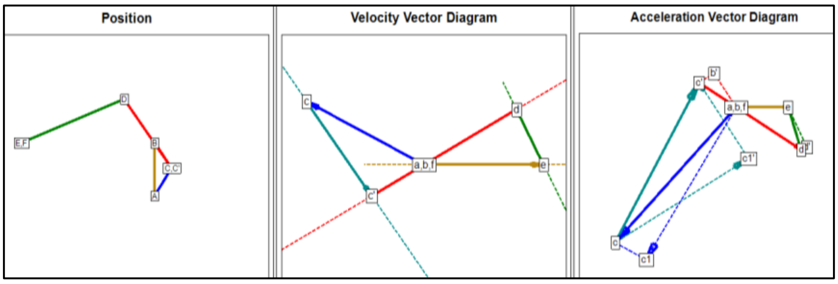
Analyses of other planar mechanisms can also be carried out by following the same procedure. Their position, velocity and acceleration vector diagrams are shown in Figure 8.



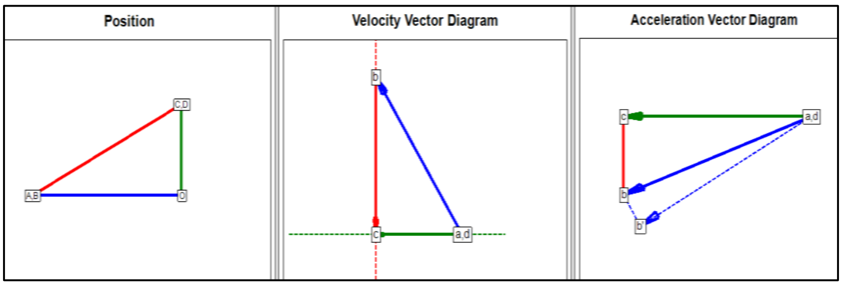
(a) Vector diagrams for four-bar mechanism



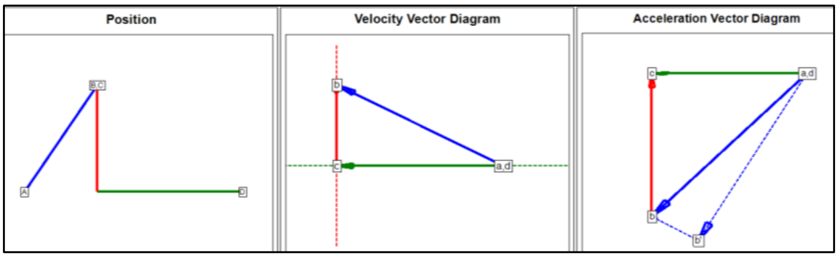
(b) Vector diagrams for crank and slotted lever mechanism



(c) Vector diagrams for Whitworth quick return mechanism



(d) Vector diagrams for elliptical trammel mechanism



(e) Vector diagrams for scotch-yoke mechanism

Figure 8: Analysis of planar mechanisms

5. VALIDATION OF RESULTS

All mechanisms were modelled individually in Autodesk Inventor (student version) and analyses were performed using "Dynamic Simulation" module in Inventor. The results were compared at discrete points for some input parameters, and found to be matching with the results obtained in MechAnalyzer software. However, there were some minor discrepancies. The input used for both the analysis is given in Table 1.

Table 1. Input parameters for slider-crank mechanism

Link 1 (mm)	Link 2 (mm)	Link 3 (mm)	Offset (mm)	Crank Angle (deg.)	Angular velocity (rad/s)	Angular Acceleration (rad/s^2)
230	60	140	0	116.90	11.59	21.15

For validation, results were compared at several discrete points but for the sake of brevity, only one comparison table is shown in Table 2. The values closely match thus validating the results of the VAD module of MechAnalyzer software.

Table 2. Comparison of results for slider crank mechanism

Results / Software	Velocity of slider (mm/s)	Acceleration of slider (mm/$\mathbf{s}^2$)	Angle of Link 2 (BC) (deg)	Angular velocity of Link 2 (BC) (rad/s)	Angular acceleration of Link 2 (BC) (rad/$\mathbf{s}^2$)
VAD Module of MechAnalyzer	489.96	4827.11	337.53	2.43	57.54
Autodesk Inventor	489.96	4827.12	337.53	2.43	57.55

6. FUTURE SCOPE

MechAnalyzer has been developed since 2013 and is already loaded with several good features that can be readily used while teaching courses related to Theory of Machines. Along with analytical formulations for kinematic and dynamic analyses, formulations based on graphical method will be further explored to include modules on:

- Static force analysis
- Balancing of rotating and reciprocating masses

7. CONCLUSIONS

Velocity and Acceleration Diagrams (VAD) module as a part of MechAnalyzer, a 3D model based mechanism learning software has been presented here. It takes link's length, velocity, and acceleration values of the driving link as input and draws position, velocity, and acceleration diagrams for common planar mechanisms. The animation of drawing the vectors can also be visualised in a way they would be drawn by a teacher on the board. Teachers using this software can minimize drawing vector diagrams on board. Students can learn mechanism concepts through graphical method effectively by visualizing the animation of vector diagrams. The VAD module is freely available with MechAnalyzer Version 4 for academic use. It can be downloaded from www.roboanalyzer.com/mechanalyzer.html.

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